DIFFERENTIAL GEOMETRY MID TERM EXAM

This exam is of **30 marks** and is **3 hours long** - from 10 am to 1pm. Please **read all the questions carefully**. Please feel free to use whatever theorems you have learned in class after stating them clearly.

I have not used any unfair or illegal means to answer any of the questions in this exam.

Name: **Signature:**

1, Recall that the **Normal Plane** at a point P is the plane spanned by $\mathbf{N}((P)$ and $\mathbf{B}(P)$. Prove that if all normal planes of a curve pass through a particular point then the curve lies on a sphere.   (5)

2. Calculate the Frenet frame $\mathbf{T}, \mathbf{N}, \mathbf{B}$, the curvature κ and the torsion τ for the following curve   (10)
$$\alpha(t) = \left(t, t^2/2, t\sqrt{1+t^2} + \log(t + \sqrt{1+t^2})\right)$$

3. Compute the following for the surface given by the parametrization
$$x(u,v)- = (a + b\cos(u)\cos(v), (a + b\cos(u))\sin(v), b\sin(u)) \, (0 < b < a)$$

   - I_P (2)
   - II_P (2)
   - Matrix of the shape operator S_P (2)
   - The surface area in the region $0 \le u, v \le 2\pi$ (5)
   - The mean curvature H (2)
   - The Gaussian curvature K.   (2)

4. Show that a **ruled surface** has Gaussian curvature $K \le 0$.   (5)